Have shown: $n \geq 3$. Then $\mathcal{M}_n[\frac{1}{6}]/\mathbb{Z}[\frac{1}{6n}]$ representable

Would like to show: $\mathcal{M}_n/\mathbb{Z}[\frac{1}{n}]$ representable.

We will follow Igusa <u>Construction</u>:

① Show by hand that $\mathcal{M}_3/\mathbb{Z}[\frac{1}{3}]$, $\mathcal{M}_4/\mathbb{Z}[\frac{1}{2}]$ are representable. (We only do $\mathcal{M}_3$.)

This requires us to also speak about the Weil pairing $E[n] \times E^\vee[n] \longrightarrow \mu_n$.

② Given $n \geq 3$, we then consider diagram

$$\begin{tikzcd}
 & \mathcal{M}_{3n} \arrow[dl, "q"'] \arrow[dr] & \\
\underbrace{\ker\left(GL_2(\mathbb{Z}/3n) \longrightarrow GL_2(\mathbb{Z}/n)\right)}_{G} \backslash \mathcal{M}_{3n} & & \mathcal{M}_n[\frac{1}{3}]
\end{tikzcd}$$

and prove that the quotient & $\mathcal{M}_n[\frac{1}{3}]$ are isomorphic. Similarly for $\mathcal{M}_{4n} \longrightarrow \mathcal{M}_n[\frac{1}{2}]$.

Then $\mathcal{M}_n[\frac{1}{2}]$ & $\mathcal{M}_n[\frac{1}{3}]$ glue to $\mathcal{M}_n$.

In principle, ② is same as our quotient construction from $\widetilde{M}_n$. However, the G-torsor $\mathcal{F}$ is only trivial étale locally. Thus we will develop some descent statements for ECs.